10th Class 2017

Math (Science)	Group-I	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Write the name of any two methods for solving a quadratic equation.

Ans The name of any two methods for solving a quadratic equation are:

1. Factorization Method.
2. Completing Square Method.

(ii) Solve: $x^2 + 2x - 2 = 0$

Ans Here, $a = 1$, $b = 2$, $c = -2$

We may solve the above equation through quadratic formula, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 + 8}}{2}$$

$$= \frac{-2 \pm \sqrt{12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{3}}{2}$$

$$= -1 \pm \sqrt{3}$$

(iii) Evaluate: $(1 - 3w - 3w^2)^5$

Ans Given:

$$(1 - 3w - 3w^2)^5$$

By taking common, we get

$$= [1 - 3(w + w^2)]^5$$

As we know that: $w + w^2 = -1$

$= [1 - 3(-1)]^5$
$= (1 + 3)^5$
$= 4^5$
$= 1024$

(iv) Evaluate: $\omega^{37} + \omega^{38} - 5$.

Ans Given:

$\omega^{37} + \omega^{38} - 5$
$= \omega^{36}.\omega + \omega^{36}.\omega^2 - 5$
$= \omega^{36}(\omega + \omega^2) - 5$
$= (\omega^3)^{12}(-1) - 5$
$= -1 - 5$
$= -6$

(v) Without solving find the sum and the product of roots of quadratic equation: $3x^2 + 7x - 11 = 0$.

Ans Here, $a = 3, b = 7, c = -11$

Sum of the roots:

$$S = \alpha + \beta = \frac{-b}{a} = \frac{-7}{3}$$

Product of the roots:

$$P = \alpha\beta = \frac{c}{a} = \frac{-11}{3}$$

(vi) Write the quadratic equation having the roots: $-1, -7$.

Ans Sum of the roots:

$\alpha + \beta = -1 + (-7) = -8$

Product of the roots:

$\alpha\beta = (-1)(-7) = 7$

Thus the quadratic equation will be:

$x^2 - Sx + P = 0$
$x^2 - (-8)x + 7 = 0$
$x^2 + 8x + 7 = 0$

(vii) Define direct variation.

Ans If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity, then this variation is called direct variation.

(viii) **Find the fourth proportional to 8, 7, 6.**

Ans Let the fourth proportional is x:

$$8 : 7 :: 6 : x$$

$$8 \times x = 7 \times 6$$

$$x = \frac{42}{8}$$

$$x = \frac{21}{4}$$

(ix) **Find x if 6 : x : : 3 : 5.**

Ans

$$x \times 3 = 6 \times 5$$

$$x = \frac{6 \times 5}{3}$$

$$x = \frac{30}{3}$$

$$\boxed{x = 10}$$

3. Write short answers to any SIX (6) questions: (12)

(i) **Define a rational fraction.**

Ans An expression of the form $\frac{N(x)}{D(x)}$, where N(x) and D(x) are polynomials in x with real coefficients and $D(x) \neq 0$, is called a rational fraction.

(ii) **Resolve $\frac{1}{x^2 - 1}$ into partial fraction.**

Ans

$$\frac{1}{x^2 - 1} = \frac{1}{(x + 1)(x - 1)}$$

$$\frac{1}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1}$$

$$1 = \frac{A}{(x + 1)}(x^2 - 1) + \frac{B}{(x - 1)}(x^2 - 1)$$

$$1 = A(x - 1) + B(x + 1) \quad \text{(i)}$$

Put $x = 1$ in (i)

$$1 = A(1 - 1) + B(1 + 1)$$

$$1 = 0 + 2B$$
$$2B = 1$$
$$\boxed{B = \frac{1}{2}}$$

Similarly, put $x = -1$ in (i),

$$1 = A(-1 - 1) + B(-1 + 1)$$
$$1 = A(-2) + 0$$
$$1 = -2A$$
$$\Rightarrow \quad -2A = 1$$
$$\boxed{A = \frac{-1}{2}}$$

Finally, by putting the values in (i), we have

$$\frac{1}{x^2 - 1} = \frac{1}{2(x - 1)} - \frac{1}{2(x + 1)}$$

(iii) Define subset.

Ans If A and B are two sets and every element of A is a member of B, then A is called subset of B.

(iv) If L = {a, b, c}, M = {3, 4}, then find L × M.

Ans $L \times M = \{a, b, c\} \times \{3, 4\}$

$L \times M = \{(a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)\}$

(v) Find domain and range of the binary relation, R = {(1, 1), (2, 2), (3, 3), (4, 4)}.

Ans Dom R = {1, 2, 3, 4}

Range R = {1, 2, 3, 4}

(vi) If (2a + 5, 3) = (7, b – 4), find a, b.

Ans By comparing the values, we get

$$2a + 5 = 7 \quad ; \quad 3 = b - 4$$
$$2a = 7 - 5 \quad ; \quad 3 + 4 = b$$
$$2a = 2 \quad ; \quad 7 = b$$
$$a = \frac{2}{2} \quad ; \quad \Rightarrow \boxed{b = 7}$$
$$\boxed{a = 1}$$

Thus, {a = 1, b = 7}.

(vii) **Write two properties of arithmetic mean.**

Ans Two properties of arithmetic mean are:

1. Mean is affected by change in origin.
2. Sum of the deviations of the variable X from its mean is always zero.

(viii) **Define mode.**

Ans Mode is defined as the most frequent value in the data.

(ix) **The sugar contents for a random sample of 6 packs of juices of a certain brand are found to be 2.3, 2.7, 2.5, 2.9, 3.1 and 1.9 milligram, find the median.**

Ans Arranging the values by increasing order 1.9, 2.3, 2.5, 2.7, 2.9, 3.1.

$$\text{Median} = \frac{1}{2}[\text{size of } (3^{rd} + 4^{th}) \text{ values}]$$

$$= \frac{2.5 + 2.7}{2}$$

$$= 2.6 \text{ Milligram}$$

---

4. Write short answers to any SIX (6) questions: (12)

(i) **Define radian measure of an angle.**

Ans The angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle is called one Radian.

(ii) **Convert 15° to radian.**

Ans

$$15^\circ = 15 \times \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{12} \text{ radian}$$

(iii) **Find 'r', when $l = 56$ cm, $\theta = 45^\circ$.**

Ans $l = 56$ cm, $\theta = 45^\circ$, $r = ?$

By converting the θ into radians,

$$45^\circ = 45 \times \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{4} \text{ radians}$$

We have,

$$l = r\,\theta$$

$$\Rightarrow \quad r = \frac{l}{\theta}$$

$$= \frac{56}{\frac{\pi}{4}}$$

$$= \frac{56 \times 4}{\pi}$$

$$r = 71.27 \text{ cm}$$

(iv) What is meant by zero dimension?

Ans Projection of a vertical line segment $\overline{CD}$ on a line segment $\overline{AB}$ is a point on $\overline{AB}$ which is of zero dimension.

(v) Define chord of a circle.

Ans The joining of any two points on the circumference of the circle is called chord of a circle.

(vi) Define tangent to a circle.

Ans A tangent to a circle is the straight line which touches the circumference at one point only.

(vii) What is meant by sector of a circle?

Ans The sector of a circle is an area bounded by any two radii and the arc intercepted between them.

(viii) Define circumangle.

Ans A circumangle is subtended between any two chords of a circle, having common point on its circumference.

(ix) Define inscribed circle.

Ans A circle which touches the three sides of a triangle internally is known as inscribed circle.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation by completing square: (4)

$$11x^2 - 34x + 3 = 0$$

Ans

$$11x^2 - 34x = -3$$

$$x^2 - \frac{34}{11}x = \frac{-3}{11}$$

Adding $\left(\frac{17}{11}\right)^2$ on both sides,

$$x^2 - 2(x)\left(\frac{17}{11}\right) + \left(\frac{17}{11}\right)^2 = \frac{-3}{11} + \left(\frac{17}{11}\right)^2$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{-3}{11} + \frac{289}{121}$$

$$= \frac{-33 + 289}{121}$$

$$= \frac{256}{121}$$

Taking square root on both sides, we have

$$x - \frac{17}{11} = \pm\frac{16}{11}$$

$$x = \frac{17}{11} \pm \frac{16}{11}$$

$$x = \frac{17}{11} + \frac{16}{11} \quad ; \quad x = \frac{17}{11} - \frac{16}{11}$$

$$= \frac{17 + 16}{11} \quad ; \quad = \frac{17 - 16}{11}$$

$$= \frac{33}{11} \quad ; \quad x = \frac{1}{11}$$

$$x = 3$$

(b) If α, β are the roots of equation $lx^2 + mx + n = 0$, $(l \neq 0)$, then find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. (4)

Ans $a = l, \quad b = m, \quad c = n$

$$\alpha + \beta = \frac{-b}{a} = \frac{-m}{l}$$

$$\alpha\beta = \frac{c}{a} = \frac{n}{l}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(\frac{-m}{l}\right)^2 - 2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2}$$

$$= \frac{\frac{m^2}{l^2} - \frac{2n}{l}}{\frac{n^2}{l^2}}$$

$$= \frac{\frac{m^2 - 2ln}{l^2}}{\frac{n^2}{l^2}}$$

$$= \frac{1}{n^2}(m^2 - 2ln)$$

Q.6.(a) Using theorem of componendo-dividendo find the value of: $\frac{x + 2y}{x - 2y} + \frac{x + 2z}{x - 2z}$ if $x = \frac{4yz}{y + z}$. (4)

Ans

$$x = \frac{4yz}{y + z}$$

$$\frac{x}{2y} = \frac{2z}{y + z}$$

$$\frac{x+2y}{x-2y} = \frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y} = \frac{y+3z}{z-y} \quad (1)$$

Similarly,

$$\frac{x}{2z} = \frac{2y}{y+z}$$

$$\frac{x+2z}{x-2z} = \frac{2y+y+z}{2y-y-z}$$

$$= \frac{3y+z}{y-z}$$

$$= -\left(\frac{3y+z}{z-y}\right)$$

$$\frac{x+2z}{x-2z} = \frac{-3y-z}{z-y} \quad (2)$$

From (1) and (2), we have

$$\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z} = \frac{y+3z}{z-y} + \frac{-3y-z}{z-y}$$

$$= \frac{y+3z-3y-z}{z-y}$$

$$= \frac{2z-2y}{z-y}$$

$$= \frac{2(z-y)}{z-y}$$

$$= 2$$

(b) Resolve into partial fractions: $\frac{x-11}{(x-4)(x+3)}$. **(4)**

Ans

$$\frac{x-11}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$$

$$x - 11 = A(x+3) + B(x-4) \quad (i)$$

Put $x = 4$, $x = -3$ in (i)

Firstly, $4 - 11 = A(4+3) + B(4-4)$

$$-7 = A(7) + 0$$

$\Rightarrow \quad 7A = -7$

$$\boxed{A = -1}$$

And

$$-3 - 11 = A(-3 + 3) + B(-3 - 4)$$
$$-14 = 0 + B(-7)$$

$\Rightarrow \quad -7B = -14$

$$\boxed{B = 2}$$

So,

$$\frac{x - 11}{(x - 4)(x + 3)} = \frac{-1}{x - 4} + \frac{2}{x + 3}$$

Q.7.(a) If U = {1, 2, 3, -----, 10}, A = {1, 3, 5, 7, 9}, B = {1, 4, 7, 10}, then verify that $A - B = A \cap B'$. (4)

Ans L.H.S = A – B

= {1, 3, 5, 7, 9} – {1, 4, 7, 10}

= {3, 5, 9} (i)

B′ = U – B

= {1, 2, 3, -----, 10} – {1, 4, 7, 10}

= {2, 3, 5, 6, 8, 9}

R.H.S = $A \cap B'$

= {1, 3, 5, 7, 9} ∩ {2, 3, 5, 6, 8, 9}

= {3, 5, 9} (ii)

So, proved from (i) and (ii),

L.H.S = R.H.S

$A - B = A \cap B'$

(b) Calculate the variance for the data: (4)

10, 8, 9, 7, 5, 12, 8, 6, 8, 2

Ans

X	X^2
10	100
8	64
9	81
7	49
5	25
12	144
8	64

6	36
8	64
2	4
75	631

Here, $\Sigma X = 75$, $\Sigma X^2 = 631$, $n = 10$

$$\text{Variance} = S^2 = \frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2$$

$$= \frac{631}{10} - \left(\frac{75}{10}\right)^2$$

$$= 63.1 - 56.25$$

$$S^2 = 6.85$$

Q.8.(a) Prove that: $\sin\theta(\tan\theta + \cot\theta) = \sec\theta$. (4)

Ans L.H.S $= \sin\theta(\tan\theta + \cot\theta)$

$$= \sin\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \sin\theta\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \sin\theta\left(\frac{1}{\cos\theta\sin\theta}\right)$$

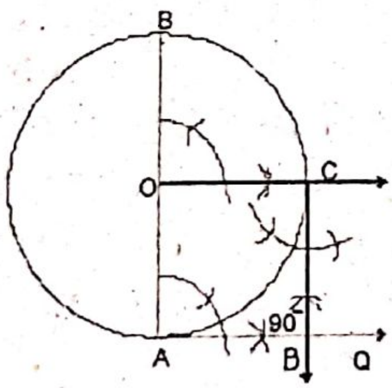
$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

= R.H.S Proved

(b) Draw two perpendicular tangents to a circle of radius 3 cm.

Ans



Step of Construction:

Steps:

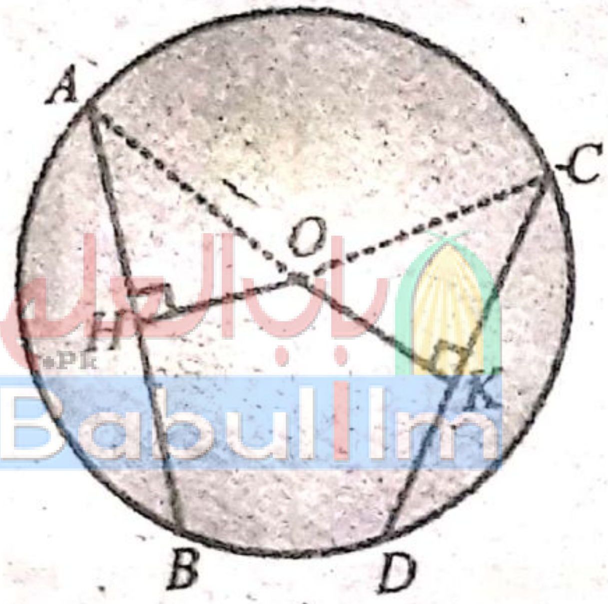
1. Take a point O.
2. Take O as centre and draw circle of radius 3 cm.
3. Take AOB any diameter of the circle.
4. Draw $m\angle BOC = 90^\circ$, $m\angle AOC = 90^\circ$.
5. Draw tangents at point A, C. These are $\overrightarrow{CP}$, $\overrightarrow{AQ}$.

Result:

$\overrightarrow{AQ}$, $\overrightarrow{CP}$ are required tangents at point D at 90°.

Q.9. Prove that if two chords of a circle are congruent, then they will be equidistant from the centre. (4)

Ans



Given:

$\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with centre at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

To prove:

$m\overline{OH} = m\overline{OK}$

Construction:

Join O with A and O with C.

So that we have $\angle rt\Delta^s$ OAH and OCK.

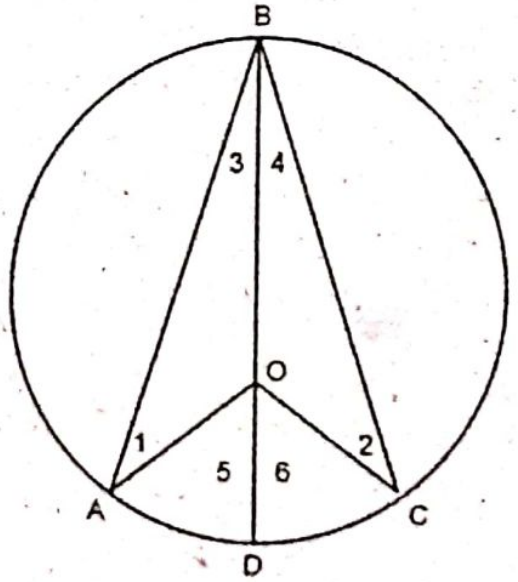
Proof:

Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	$\overline{OH} \perp \overline{AB}$ By Theorem 3
i.e., $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (i)	
Similarly, $\overline{OK}$ bisects chord $\overline{CD}$	$\overline{OK} \perp \overline{CD}$ By Theorem 3
i.e., $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (ii)	
But $m\overline{AB} = m\overline{CD}$ (iii)	Given
Hence, $m\overline{AH} = m\overline{CK}$ (iv)	Using (i), (ii) & (iii)
Now in $\angle$rt Δ^s OAH $\leftrightarrow$ OCK	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp $\overline{OA}$ = hyp $\overline{OC}$	Radii of the same circle
$m\overline{AH} = \overline{CK}$	Already proved in (iv)
$\therefore$ $\Delta OAH \cong \Delta OCK$	H.S postulate
$\Rightarrow$ $m\overline{OH} = m\overline{OK}$	

OR

Prove that the measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

Ans



Given:

$\overset{\frown}{AC}$ is an arc of a circle with center O; whereas $\angle AOC$ is the central angle and $\angle ABC$ is circumangle.

To prove:

$m\angle AOC = 2m\angle ABC$

Construction:

Join B with O and produce it to meet the circle at D.

Write angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ as shown in the figure.

Proof:

Statements	Reasons
As $m\angle 1 = m\angle 3$ (i)	Angles opposite to equal sides in ΔOAB.
and $m\angle 2 = m\angle 4$ (ii)	Angles opposite to equal sides in ΔOBC.
Now $m\angle 5 = m\angle 1 + m\angle 3$ (iii)	External angle is the sum of internal opposite angles.
Similarly, $M\angle 6 = m\angle 2 + m\angle 4$ (iv)	
Again $M\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3$ (v)	Using (i) and (iii)
And $m\angle 6 = m\angle 4 + m\angle 4 = 2m\angle 4$ (vi)	Using (ii) and (iv)
$\Rightarrow m\angle 5 + m\angle 6 = 2m\angle 3 + m\angle 4$	Adding (v) and (vi)
$\Rightarrow m\angle AOC = 2(m\angle 3 + m\angle 4)$	
$= 2m\angle ABC$	